Strength formulae of a composite under longitudinal compression

Zheng-Ming Huang

huangzm@tongji.edu.cn

School of Aerospace Engineering & Applied Mechanics, Tongji University, China

Highlight

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1. Introduction
2. Homogenized stresses
3. True stresses
4. Physics based failure criterion
5. Fiber misalignment angle increment
6. Longitudinal compressive strength
7. Compressive strength of any composite
8. Conclusion

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1. Introduction

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One of the most difficult strengths for a composite to be predicted is the longitudinal compressive one.

This is mainly because a key parameter, i.e., an initial fiber misalignment angle necessary for analyzing a longitudinal compressive failure, is difficult to measure.

Even though initial fiber misalignment angle is assumed, a reasonable analysis for a fiber kinking failure is not easy to achieve.

In processing a composite, a fiber misalignment is almost inevitable, and a longitudinal compression induces a shear stress component in a misaligned coordinate system.

1. Introduction

The shear stress component will generate a misalignment angle increment before the composite failure.

In the misaligned coordinate system defined by the overall mis-angle (equal to the initial plus the increment), the shear stress component will cause the matrix to fail, while the axial one brings the fiber to a failure.

Whichever occurs first corresponds to the longitudinal compressive strength of the composite.

Two issues must be addressed before the longitudinal compressive strength is evaluated. One is the fiber mis-alignment angle increment, and another is matrix true stresses.

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2. Homogenized st

In mechanics of continuum media, a stress at a point is defined as averaged one of those on an infinitesimally small element containing the point through

$$\sigma_i = \left(\int_{V'} \tilde{\sigma}_i dV\right) / V', \quad V' \to 0$$

For a composite, such an element (called RVE) cannot be infinitesimal, since both the fiber and matrix must be contained in it, leading to

$$\sigma_i = \frac{1}{V'}\int_{V'} \tilde{\sigma}_i dV' = \frac{V_f'}{V'}\left(\frac{1}{V_f'}\int_{V_f'} \tilde{\sigma}_i dV'\right) + \frac{V_m'}{V'}\left(\frac{1}{V_m'}\int_{V_m'} \tilde{\sigma}_i dV'\right) = V_f \sigma_i^f + V_m \sigma_i^m$$

So, no rigorous stress of a composite exists.

2. Homogenized st

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Using a bridging equation, $\{\sigma_i^m\} = [a_{ij}]\{\sigma_j^f\}$, one obtains

$$\{\sigma_i^f\} = (V_f[I] + V_m[a_{ij}])^{-1}\{\sigma_j\}$$

$$\{\sigma_i^m\} = [a_{ij}](V_f[I] + V_m[a_{ij}])^{-1}\{\sigma_j\}$$

Although the bridging tensor $[a_{ij}]$ can be also determined by another micromechanics model, Bridging Model shows incomparable advantages over any other models.

First, Bridging Model gives, perhaps, simplest expressions for all of the bridging tensor elements, leading to closed-form formulae for internal fiber and matrix stresses:

2. Homogenized st

$$\sigma_{11}^{f}=\frac{\sigma_{11}}{V_f+V_m a_{11}}-\frac{V_m a_{12}(\sigma_{22}+\sigma_{33})}{(V_f+V_m a_{11})(V_f+V_m a_{22})}$$

$$\sigma_{11}^{m}=\frac{a_{11}\sigma_{11}}{V_f+V_m a_{11}}+\frac{V_f a_{12}(\sigma_{22}+\sigma_{33})}{(V_f+V_m a_{11})(V_f+V_m a_{22})}$$

$$\sigma_{ij}^{f}=\frac{\sigma_{ij}}{V_f+V_m a_{22}},\quad \sigma_{ij}^{m}=\frac{a_{22}\sigma_{ij}}{V_f+V_m a_{22}},$$ ***ij*=22, 33, 23**

$$\sigma_{ij}^{f}=\frac{\sigma_{ij}}{V_f+V_m a_{66}},\quad \sigma_{ij}^{m}=\frac{a_{66}\sigma_{ij}}{V_f+V_m a_{66}},$$ ***ij*=13, 12**

$$a_{11}=E^{m}/E_{11}^{f}\qquad a_{22}=a_{33}=a_{44}=0.3+0.7E^{m}/E_{22}^{f}$$

$$a_{12}=a_{13}=(S_{12}^{f}-S_{12}^{m})(a_{11}-a_{22})/(S_{11}^{f}-S_{11}^{m})$$

$$a_{55}=a_{66}=0.3+0.7G^{m}/G_{12}^{f}$$

in which, $\{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{13}, \sigma_{12}\}$ are the arbitrary stress vector applied on the composite. All of the other bridging tensor elements are zero.

Second, Bridging Model is overall more accurate than most other micromechanics models.

A comparison between the experiments and the predictions by 15 most well known models for the elastic properties of 9 composites used in world-wide failure exercises (WWFEs) shows that Bridging Model is overall the most accurate.

All the predictions use the same input data provided by the exercise organizers.

Comparison between predictions of 15 most well known models and experiments for composites used in WWFEs

Model	N	Averaged error[a] (%)	Error ratio	Rank
Bridging model	45	10.38	1.0	1
FE-square	45	13.08	1.26	2
Double inclusion method	45	13.6	1.31	3
Chamis model	45	14.09	1.36	4
Hill–Hashin-C-L model	33	17.22	1.66	5
FE-random	45	17.57	1.69	6
Generalized self-consistent	45	18.14	1.75	7
FE-hexagonal	45	19.05	1.84	8
Halpin–Tsai formulae	45	19.24	1.85	9
Modified rule of mixture	45	19.35	1.86	10
Mori–Tanaka method	45	19.59	1.89	11
FE-square diagonal	45	21.48	2.07	12
Self consistent method	45	21.82	2.1	13
Rule of mixture method	45	28.4	2.74	14
Eshelby's method	45	30.72	2.96	15

2. Homogenized st

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Several other groups have also confirmed high accuracy of Bridging Model through their comparative studies.

It was reported in *Int. J. Impact Engng*. (36: 899–912, 2009) that Bridging Model was the most accurate within the six models studied. https://doi.org/10.1016/j.ijimpeng.2008.12.012

In open-access publication, Bridging Model was verified as the best among 11 models investigated. http://doi.org/10.5772/50362

Also in an article published in *Materials* (15: 5090, 2022), Bridging Model was proven to be the best among the four analytical micromechanics models. https://doi.org/10.3390/ma15155090

2. Homogenized st

Third, when a composite is subjected to an in-plane load, the internal fiber and matrix stresses obtained using 3D and 2D Bridging Model formulae are the same, but are different if any other micromechanics model is used.

This is because the other model starts with determination of the compliance tensor [S_{ij}] for the composite. Then, the bridging tensor [a_{ij}] is back calculated from the [S_{ij}]. This cannot guarantee the resulting [a_{ij}] to be always in upper triangular form.

So, 3D bridging tensor of other model has to be used, and Bridging Model consumes least amount of calculations.

2. Homogenized st

Fourth, when the matrix undergoes a plastic or rubber-like elastic deformation, the composite constitutive relations by Bridging Model are still in closed-form.

Fifth, when interface debonding occurs in between fiber and matrix, a relative slippage displacement induced from debonded fiber and matrix interfaces has been explicitly incorporated into the composite constitutive relations by Bridging Model.

For this incorporation, only transverse tensile strength of the composite is additionally required.

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3. True stresses

Sufficient evidences have shown that the elastic properties of a composite predicted by a well established model such as Bridging Model are accurate enough.

However, even using measured data to define compliance tensor [S_{ij}], a predicted strength of the composite upon the internal stresses obtained from the below bridging tensor can still be far away from a reality

$$[a_{ij}] = V_f([S_{ij}] - [S_{ij}^m])^{-1}([S_{ij}^f] - [S_{ij}])/V_m$$

as long as the internal stresses are compared with the fiber and matrix original strengths measured monolithically.

3. True stresses

This dilemma has long been recognized in the community, and more than half a century ago researchers have pointed out that in-situ constituent strengths must be employed in criteria to assess a constituent failure.

However, experiments can only measure material original strengths such as matrix tensile, compressive and shear strengths.

Nobody knows how to measure an in-situ strength of a constituent, and even how many in-situ strengths exist is not known yet.

3. True stresses

We have found that once the homogenized stresses of the constituents are converted into true values, almost all intra-laminar failures can be reasonably estimated.

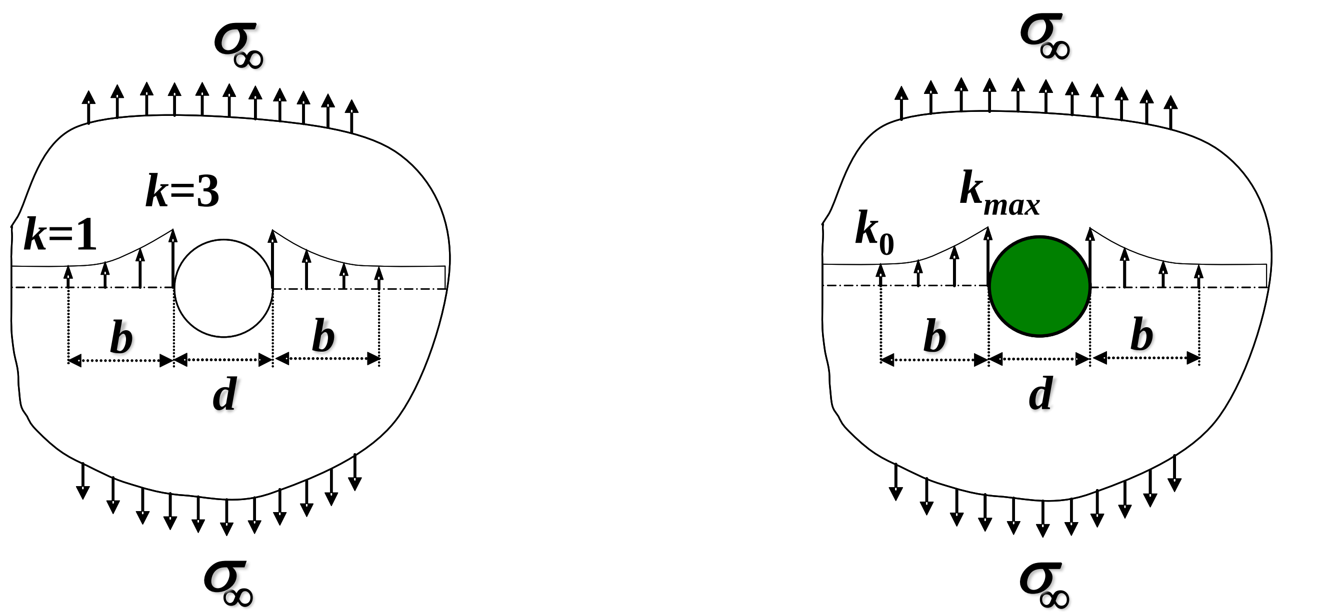
According to Eshelby, a fiber stress field is uniform no matter what kind of load is applied to the composite, and thus the fiber true and homogenized stresses are the same:

$$\{\bar{\sigma}_j^f\}_l = \{\bar{\sigma}_j^f\}_{l-1} + \{d\sigma_j^f\}$$

Matrix true stresses are obtained by multiplying its homo-genized counterparts with stress concentration factors (SCFs) of the matrix in the composite.

3. True stresses

A plate with a hole generates a stress concentration when subjected to an in-plane tension.



When the hole is filled with a fiber different from matrix in properties, stress concentrations occur as well.

3. True stresses

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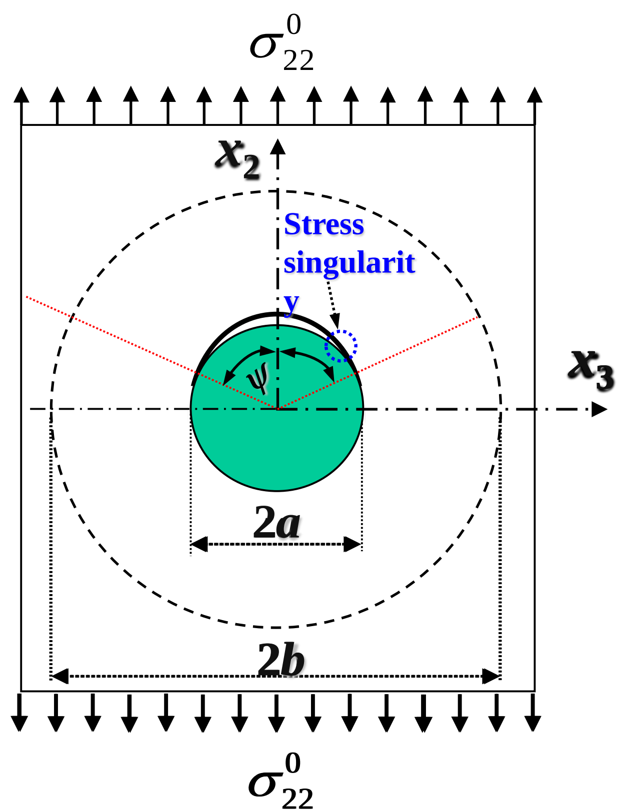
A matrix SCF cannot be defined following a classical way. Otherwise, if an interface crack occurs, matrix stresses at crack tip are singular, and the classical method would give an infinite SCF. We need to find a new definition.



3. True stresses

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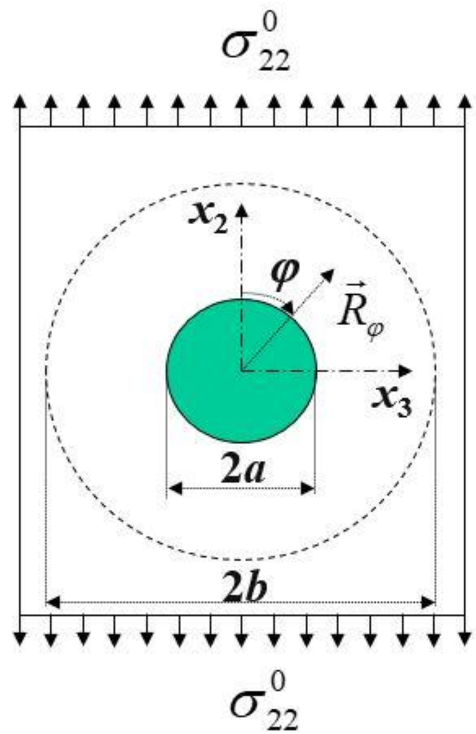
A matrix SCF cannot be defined following a classical way. Otherwise, if an interface crack occurs, matrix stresses at crack tip are singular, and the classical method would give an infinite SCF. We need to find a new definition.

As classical definition for an SCF is "point-wise stress (0D) over external one (averaged *w.r.t.* surface where the stress is applied, i.e., a 2D quantity)", a matrix SCF can only be defined as "a line-averaged stress (1D) over volume-averaged one (3D)".



The line-averaging should be along outward normal to the failure surface.

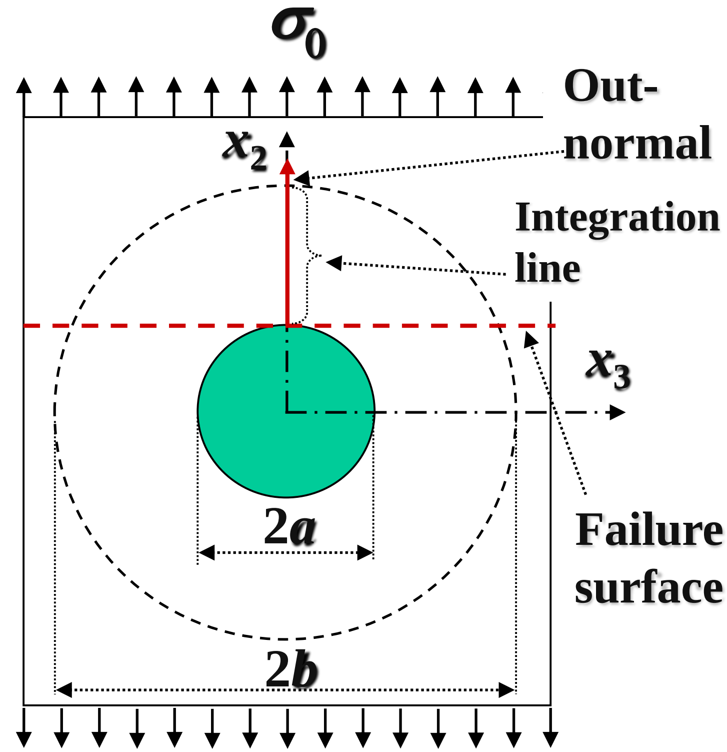
3. True stresses

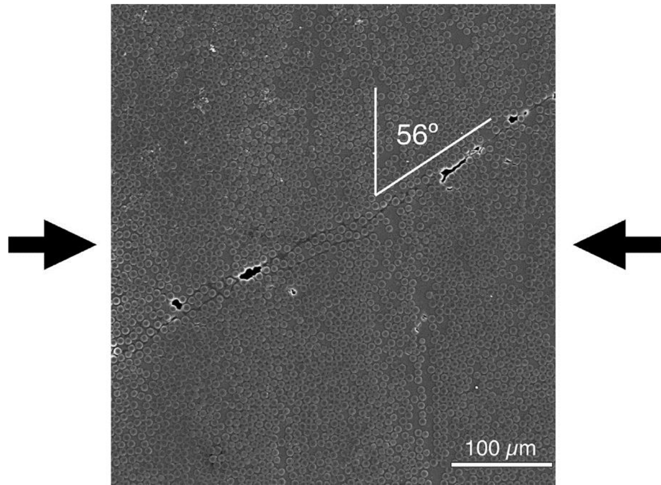
For instance, a transverse SCF of the matrix is defined as

$$K_{22}(\varphi)=\frac{1}{\left|\vec{R}_{\varphi}^{b}-\vec{R}_{\varphi}^{a}\right|}\int_{\left|\vec{R}_{\varphi}^{a}\right|}^{\left|\vec{R}_{\varphi}^{b}\right|}\frac{\tilde{\sigma}_{22}^{m}}{(\sigma_{22}^{m})_{BM}}d\left|\vec{R}_{\varphi}\right|$$

$\tilde{\sigma}_{22}^{m}$ is a point-wise stress, $(\sigma_{22}^{m})_{BM}$ is obtained by Bridging Model, φ is angle between the normal and load.

Outward normal: same with tension;





3. True stresses

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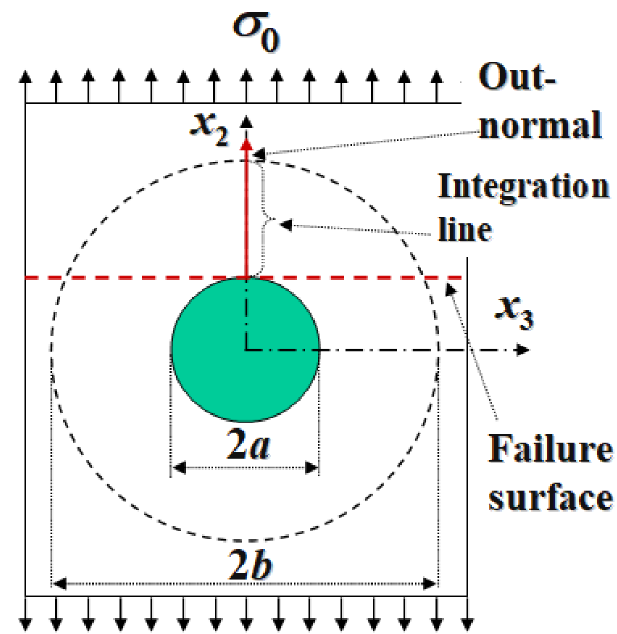
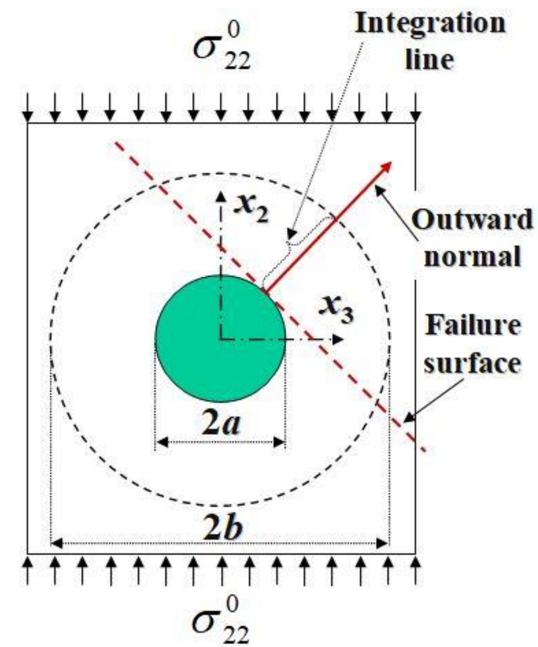
$\tilde{\sigma}_{22}^{m}$ is a point-wise stress, $(\sigma_{22}^{m})_{BM}$ is obtained by Bridging Model, φ is angle between the normal and load.

Outward normal: same with tension; in inclination with compression.





3. True stresses

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Two quantities below must be known in advance before a matrix SCF is derived:

(1) the matrix stresses by elasticity solution in coaxial cylinder assemblage (CCA) model subjected to the load;

(2) position of the outward normal to the failure surface of the composite in the CCA model under the load.

The matrix stress field in a CCA model under almost any single load with either perfect or debonded interface can be found in the literature, while the second quantity can be determined through experimental observations.

3. True stresses

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Therefore, almost all possible SCFs of the matrix in a composite have been successfully derived.

Let $\{d\sigma_i^m\}=\{d\sigma_{11}^m, d\sigma_{22}^m, d\sigma_{33}^m, d\sigma_{23}^m, d\sigma_{13}^m, d\sigma_{12}^m\}^T$ **be the matrix homogenized stress increments determined by any but preferably by Bridging Model, the overall true stresses of the matrix at the current load step are given by**

$$\{\bar{\sigma}_i^m\}_l=\{\bar{\sigma}_{11}^m, \bar{\sigma}_{22}^m, \bar{\sigma}_{33}^m, \bar{\sigma}_{23}^m, \bar{\sigma}_{13}^m, \bar{\sigma}_{12}^m\}_l^T=\{\bar{\sigma}_i^m\}_{l-1}+\{d\bar{\sigma}_i^m\}$$

$$\{d\bar{\sigma}_i^m\}=\{K_{11}d\sigma_{11}^m, K_{22}d\sigma_{22}^m, K_{33}d\sigma_{33}^m, K_{23}d\sigma_{23}^m, \bar{K}_{12}d\sigma_{13}^m, \bar{K}_{12}d\sigma_{12}^m\}^T$$

The various matrix SCFs are defined as follows.

3. True stresses

$$K_{11} = \begin{cases} 1, \ for\ a\ continuous\ \ fiber\ composite \\ K_{11}^{t}, \ for\ a\ short\ \ fiber\ composite\ with\ d\sigma_{11}^{m} > 0 \\ K_{11}^{c}, \ for\ a\ short\ \ fiber\ composite\ with\ d\sigma_{11}^{m} < 0 \end{cases}$$

$$\overline{K}_{12} = \begin{cases} K_{12}, \ for\ a\ \ perfect\ interface \\ \hat{K}_{12}, \ for\ a\ debonded\ interface \end{cases}$$

$$K_{22} = \begin{cases} K_{22}^{t}, if\ \ d\sigma_{22}^{m} > 0\ \ \&\ \ with\ perfect\ interface \\ \hat{K}_{22}^{t}, if\ \ d\sigma_{22}^{m} > 0\ \ \&\ \ with\ debonded\ interface \\ K_{22}^{c}, if\ \ d\sigma_{22}^{m} < 0 \end{cases}$$

$$K_{33} = \begin{cases} K_{22}^{t}, if\ \ d\sigma_{33}^{m} > 0\ \ \&\ \ with\ perfect\ interface \\ \hat{K}_{22}^{t}, if\ \ d\sigma_{33}^{m} > 0\ \ \&\ \ with\ debonded\ interface \\ K_{22}^{c}, if\ \ d\sigma_{33}^{m} < 0 \end{cases}$$

3. True stresses

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SCFs of the matrices in the 9 material systems used in WWFEs

	E-Glass LY556	E-Glass MY750	AS4 3501-6	T300 BSL914C	IM7 8511-7	T300 PR319	AS Epoxy	S2-Glass Epoxy	G400-800 5260
K_{12}	1.52	1.491	1.424	1.43	1.475	1.51	1.449	1.5	1.483
K_{23}	3.02	2.936	1.337	2.421	2.034	2.167	1.999	2.982	2.469
K_{22}^{t}	**3.339**	**3.253**	**2.098**	**2.143**	**2.327**	**3.123**	**2.339**	**3.317**	**2.464**
K_{22}^{c}	2.249	2.181	1.469	1.57	1.761	2.035	1.743	2.172	1.732
$\hat{K}_{12}$	1.87	1.84	1.76	1.76	1.83	1.76	1.76	1.85	1.83
$\hat{K}_{22}^{t}$	**7.69**	**7.22**	**4.95**	**5.04**	**5.41**	**6.97**	**5.43**	**7.34**	**5.68**
$K_{11}^{t,I}$	3.66	3.574	3.733	3.877	4.486	4.188	3.953	3.688	4.024
K_{11}^{t}	1.598	1.675	1.516	1.588	1.965	1.844	1.636	1.726	1.720
$K_{11}^{c,I}$	3.51	3.419	3.550	3.685	4.271	3.995	3.759	3.528	3.833
K_{11}^{c}	1.504	1.577	1.427	1.496	1.860	1.757	1.545	1.631	1.630

Simply speaking, except for longitudinal strengths of a continuous fiber composite, a current prediction for any other composite strength differs from reality at least 1.4 times and at most 7.7 times.

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4. Physical failure (

Having obtained fiber and matrix true stresses, we are able to detect a composite failure, which must correspond to a fiber or matrix failure.

Any failure of fiber or matrix is assessed by comparing its true stresses directly with its original strengths.

4.1 Criteria for fiber failures

Fiber is thin, similar to a slender bar, and mainly sustains an axial load. The first strength theory in Mech. of Mater. textbook is best applicable to detect a fiber failure.

A fiber failure is attained if any of the following is fulfilled

$$\bar{\sigma}_f^3 \le -\sigma_{u,c}^f \quad \textbf{or} \quad \bar{\sigma}_f^1 \ge \sigma_{u,t}^f$$

$\bar{\sigma}_f^1$, $\bar{\sigma}_f^3$ ($\bar{\sigma}_f^1 \ge \bar{\sigma}_f^2 \ge \bar{\sigma}_f^3$) are the first, third principal stresses . $\sigma_{u,t}^f$, $\sigma_{u,c}^f$ are the fiber axial tensile, compressive strengths

4.2 Criteria for matrix failures

The loads assumed and failure modes of the matrix are much more complicated than the fiber, and various failure criteria for it have been established based on physical, mathematical and phenomenological principles.

4. Physical failure (

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Mathematically, failure locus is multifunction of stresses and strains. Expanding it into a series and retaining only low items, the expanding coefficients are determined by measured strengths under simple types of loads.

Tsai-Wu criterion is a typical of such kind.

Physics principle originates from Mohr’s geometrical expression for a stress state: plot a failure stress state into a Mohr’s circle, and the common tangents to all such failure stress circles generates a failure envelope.

4. Physical failure (

For any given 2D stress state, a necessary and sufficient condition for an isotropic material to attain a failure status is that its stress circle is in an inward tangent to the failure envelope.

Although most failure criteria have been established phenomenolocally, the choice of a criterion from establishing foundation viewpoint should obey: physical ≥ mathematical ≥ phenomenological.

In order to establish a physics criterion for a matrix compressive failure, we approximate the failure envelope using a parabola constructed from matrix compressive and shear strengths shown below.

4. Physical failure (

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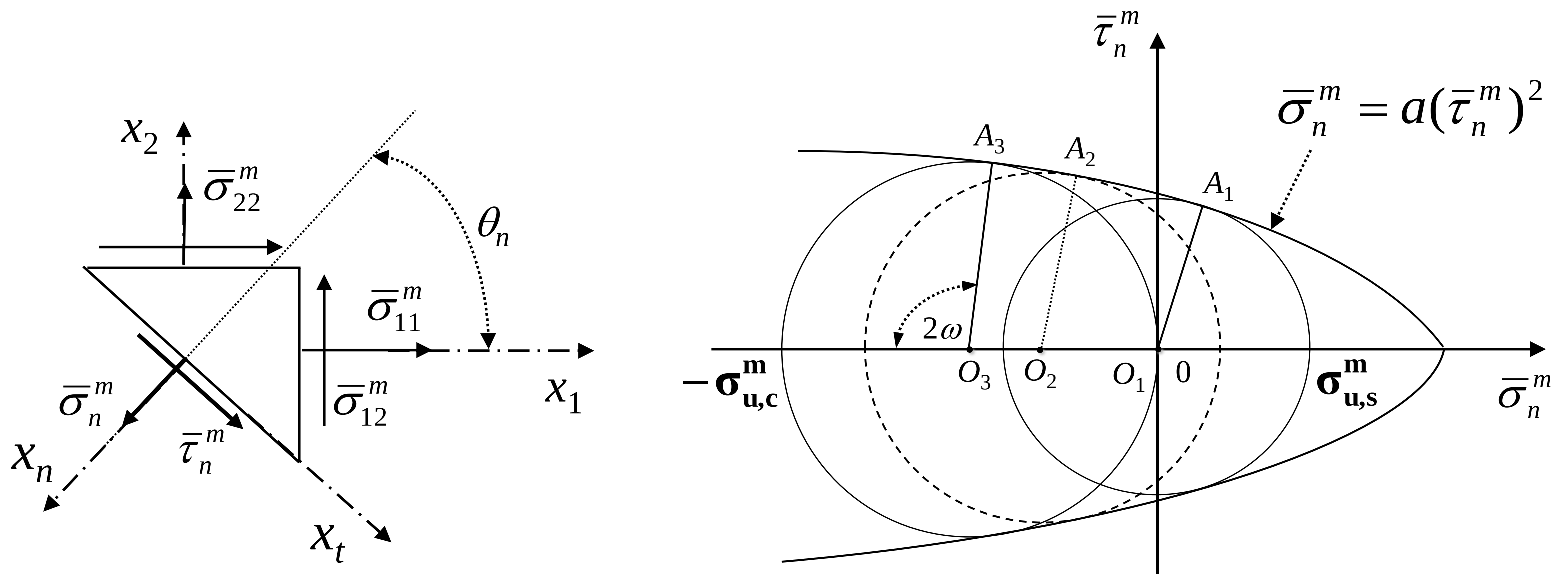

Through a simple geometrical analysis, the parabolic equation for the failure envelope is found to be

$$\bar{\sigma}_n^m = a(\bar{\tau}_n^m)^2 + b$$

$$a = \frac{2\sigma_{u,c}^m}{4(\sigma_{u,s}^m)^2 - (\sigma_{u,c}^m)^2} \qquad b = -\frac{[4(\sigma_{u,s}^m)^2 + (\sigma_{u,c}^m)^2]^2}{8\sigma_{u,c}^m[4(\sigma_{u,s}^m)^2 - (\sigma_{u,c}^m)^2]}$$

4. Physical failure (

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$\sigma_{u,c}^{m}$ and $\sigma_{u,s}^{m}$ are the matrix original compressive and shear strengths measured from monolithic material specimens.

When matrix under true stresses $(\bar{\sigma}_{11}^{m}, \bar{\sigma}_{22}^{m}, \bar{\sigma}_{12}^{m})$ dominated by compression attains failure, the stress circle must be in an inward tangent with the parabola, and the stresses at the contacting point, $(\bar{\sigma}_{11}^{m,0}, \bar{\sigma}_{22}^{m,0}, \bar{\sigma}_{12}^{m,0})$, fulfills

$$(\bar{\sigma}_{11}^{m,0}, \bar{\sigma}_{22}^{m,0}, \bar{\sigma}_{12}^{m,0}) = \delta(\bar{\sigma}_{11}^{m}, \bar{\sigma}_{22}^{m}, \bar{\sigma}_{12}^{m})$$

$$\delta = \frac{4a\bar{\sigma}_{n}^{m,1} \pm \sqrt{16a^2(\bar{\sigma}_{n}^{m,1})^2 - 16a^2r^2(1+4ab)}}{8a^2r^2}$$

$$0 < \delta \le 1$$

4. Physical failure (

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$$\bar{\sigma}_n^{m,1} = \frac{\bar{\sigma}_{11}^m + \bar{\sigma}_{22}^m}{2} \qquad r = \sqrt{0.25(\bar{\sigma}_{11}^m - \bar{\sigma}_{22}^m)^2 + (\bar{\sigma}_{12}^m)^2}$$

On the matrix failure surface, the shear stress component is given by

$$\bar{\tau}_n^m = \sqrt{\frac{(\bar{\sigma}_{11}^{m,0} + \bar{\sigma}_{22}^{m,0})a - 1 - 2ab}{2a^2}}$$

This formula is of critical importance to derive a kinking failure condition for a composite under a longitudinal compression.

The failure surface angle can be determined from

$$\frac{\bar{\sigma}_{11}^{m,0}-\bar{\sigma}_{22}^{m,0}}{2}\cos(2\theta_n)+\bar{\sigma}_{12}^{m,0}\sin(2\theta_n)=\frac{2a\bar{\sigma}_n^{m,1}\delta-1}{2a}-\frac{\bar{\sigma}_{11}^{m,0}+\bar{\sigma}_{22}^{m,0}}{2}$$

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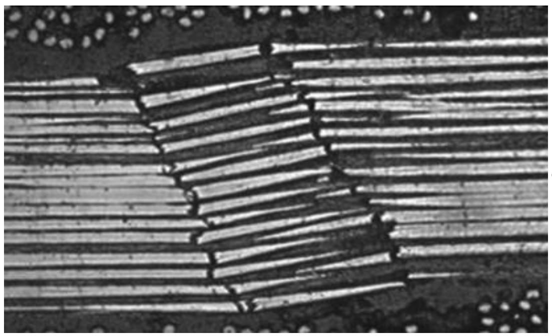
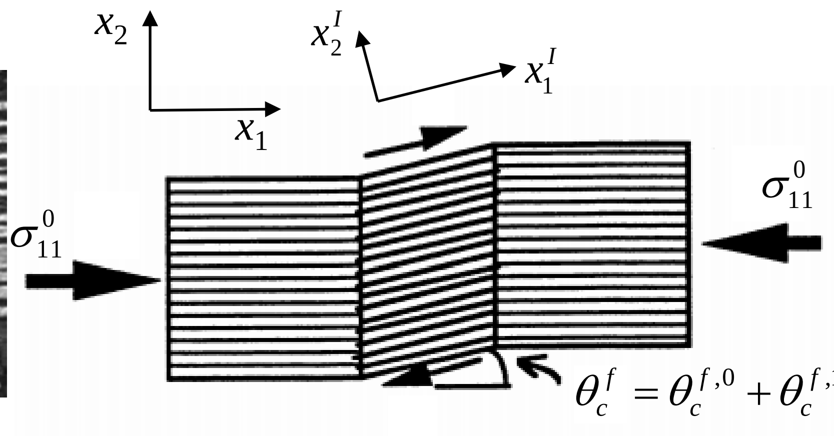
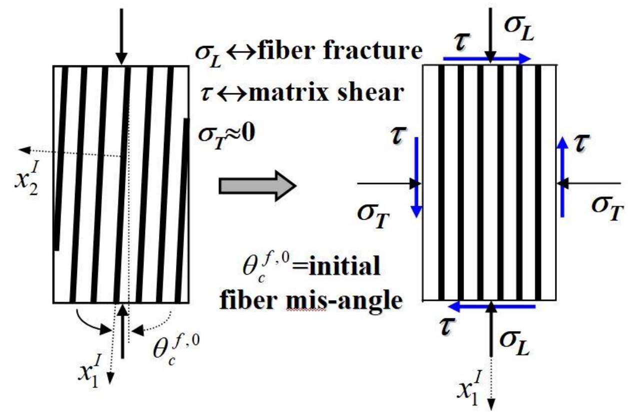
5. Fiber mis-angle i

Let composite with an initial fiber misalignment angle $\theta_c^{f,0}$ be subjected to a longitudinal compression, which can be decomposed into an axial normal, a shear and a transverse normal stress components.







The transverse stress component can be neglected due to the small mis-angle, but the shear one will amplify the angle and generate a misalignment angle increment $\theta_c^{f,1}$.

5. Fiber mis-angle i

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When the composite assumes the longitudinal compressive strength, the angle increment attains its maximum.

In the misaligned coordinate system defined by the overall angle $\theta_c^f = \theta_c^{f,0} + \theta_c^{f,1}$, any applied load $\{\sigma_{11},\sigma_{22},\sigma_{33}\}$ generates the following stress components

$$\sigma_{11}^I = \frac{\sigma_{11}+\sigma_{22}}{2} + \frac{\sigma_{11}-\sigma_{22}}{2}\cos(2\theta_c^f) + \sigma_{12}\sin(2\theta_c^f)$$

$$\sigma_{22}^I = \frac{\sigma_{11}+\sigma_{22}}{2} - \frac{\sigma_{11}-\sigma_{22}}{2}\cos(2\theta_c^f) - \sigma_{12}\sin(2\theta_c^f)$$

$$\sigma_{12}^I = -\frac{\sigma_{11}-\sigma_{22}}{2}\sin(2\theta_c^f) + \sigma_{12}\cos(2\theta_c^f)$$

In the above, σ_{11}^{I} is along the fiber axial direction. When σ_{11}<0 and σ_{22}=σ_{12}=0, the stresses in the misaligned coordinate system are resulted from a longitudinal compression.

As aforementioned, it is critical for us to derive a formula for the fiber misalignment angle increment, $\theta_c^{f,1}$.

First of all, the fiber and especially the matrix true stresses at the given load σ_{11}, $(\bar{\sigma}_{11}^{m,I}, \bar{\sigma}_{22}^{m,I}, \bar{\sigma}_{12}^{m,I})$, are calculated using Bridging Model and the matrix true stress theory.

Supposing the matrix attains failure, the shear stress on the matrix failure surface reads

5. Fiber mis-angle i

$$\bar{\tau}_n^m = \sqrt{\frac{(\bar{\sigma}_{11}^{m,I} + \bar{\sigma}_{22}^{m,I})a - 1 - 2ab}{2a^2}}$$

On the other hand, the matrix true shear stress in the composite subjected to an in-plane shear is given by

$$\bar{\sigma}_{12}^m = \frac{K_{12} a_{66} \sigma_{12}}{(V_f + V_m a_{66})}$$

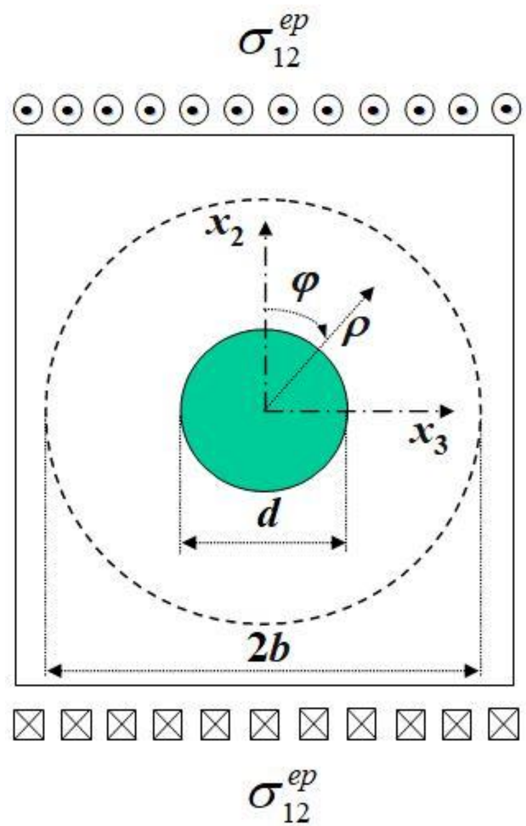
Therefore, when the matrix assumes the true shear stress on its failure surface, it is equivalent that the matrix be subjected to following in-plane shear load

$$\sigma_{12}^{eq} = \frac{(V_f + a_{66} V_m)}{a_{66} K_{12}} \sqrt{\frac{(\bar{\sigma}_{11}^{m,I} + \bar{\sigma}_{22}^{m,I})a - 1 - 2ab}{2a^2}}$$

5. Fiber mis-angle i

The matrix point-wise strain in the composite due to the equivalent shear load is found to be

$$\tilde{\gamma}_{12}^{m} = -\frac{\sigma_{12}^{ep}}{G^m}\left[1 - d^2 \frac{(G_{12}^f - G^m)(x_2{}^2 - x_3{}^2)}{4(G_{12}^f + G^m)(x_2{}^2 + x_3{}^2)^2}\right]$$



d **is the fiber diameter. It is noted that herein (x_1, x_2, x_3) ≡ (x_1^I, x_2^I, x_3^I), for simplifying an expression.**

The mis-angle increment, $\theta_c^{f,1}$, will be determined on that averaged axial displacement of the RVE be zero.

5. Fiber mis-angle i

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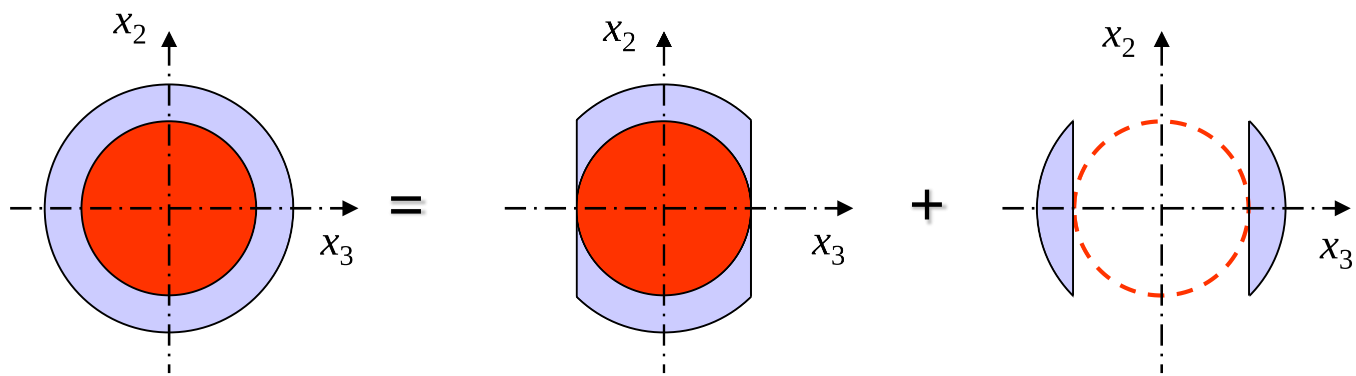
In order to evaluate the axial displacement, we subdivide the RVE into the following two parts:



The contributions from the two parts to the axial displacement are derived respectively as follows.

In the first part containing the fiber, the averaged axial displacement of a longitudinally cross-section parallel to x_2 axis is consisted of two portions.

5. Fiber mis-angle i

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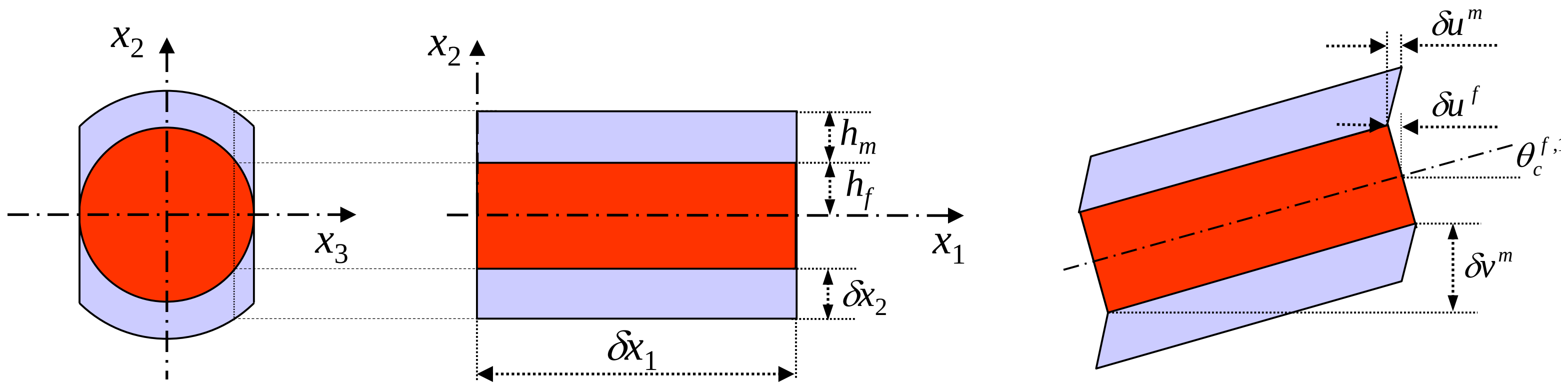
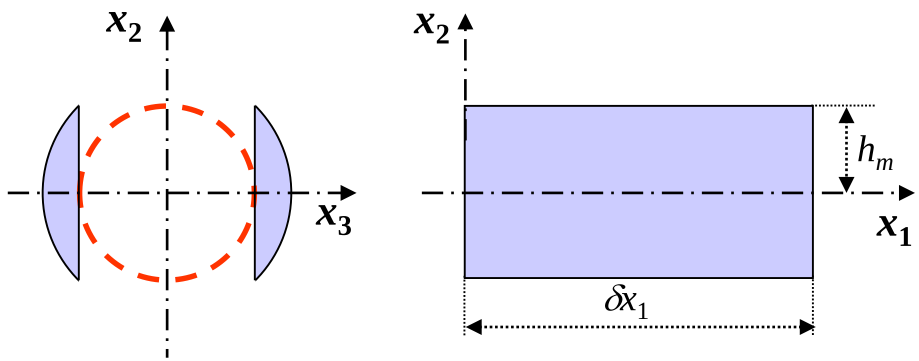
The first portion is resulted from the matrix, containing the displacements by both the shear strains and an orientation, whereas the second is from the fiber orientation:



The second part of RVE without fiber only generates shear strains, which induce an averaged axial displacement for the RVE as schematically indicated below.

5. Fiber mis-angle i



In summary, the averaged axial displacement of a longitudinally cross-sectional plane is found to be:

$$\delta u = \begin{cases} \gamma_{12}^{m}(x_3)h_m(x_3) - \theta_c^{f,1}[h_m(x_3) + h_f(x_3)], & -0.5d \le x_3 \le 0.5d \\ \gamma_{12}^{m}(x_3)h_m(x_3), & |x_3| > 0.5d \end{cases}$$

$$\gamma_{12}^{m}(x_3) = \begin{cases} \dfrac{1}{h_m(x_3)} \displaystyle\int_{\sqrt{0.25d^2 - x_3^2}}^{\sqrt{b^2 - x_3^2}} \tilde{\gamma}_{12}^{m}(x_2, x_3)dx_2, & -0.5d \le x_3 \le 0.5d \\ \dfrac{1}{h_m(x_3)} \displaystyle\int_{0}^{\sqrt{b^2 - x_3^2}} \tilde{\gamma}_{12}^{m}(x_2, x_3)dx_2, & |x_3| > 0.5d \end{cases}$$

Letting the averaged axial displacement of the RVE be zero, i.e.,

$$\delta\overline{u} = \frac{1}{2b}\int_{-b}^{b} \delta u(x_3)dx_3 = 0$$

one obtains: $$\theta_c^{f,1} = \frac{\sigma_{12}^{eq}(1-V_f)\pi}{2G^m[\sqrt{V_f}\sqrt{1-V_f} + \sin^{-1}(\sqrt{V_f})]}$$

As the equivalent shear stress on the right hand side also depends on the misalignment angle increment, an itera-tion has to be used for solution in which the angle incre-ment attains a convergence and at the same time δ≈1 is assumed in the physics failure criterion.

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6. Compre.-strengt

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Steps for evaluating a longitudinally compressive strength of a unidirectional (UD) composite are summarized below.

(1) Input parameters, including fiber elastic constants and compressive strength, matrix elastic constants, and shear and compressive strengths, fiber volume fraction and fiber initial misalignment angle.

(2) For any longitudinal compression with $\sigma_{11}<0$ and $\sigma_{22}=\sigma_{12}=0$, iteratively solve for the misalignment angle increment $\theta_c^{f,1}$ so that the following equation is fulfilled

6. Compre.-strengt

$$f = \theta_c^{f,1} - \frac{\sigma_{12}^{eq}(1-V_f)\pi}{2G^m[\sqrt{V_f}\sqrt{1-V_f} + \sin^{-1}(\sqrt{V_f})]} \approx 0$$

The solution can be achieved through bisection highlighted as follows.

- **Giving $\theta_c^{f,1}$, calculate stresses $(\sigma_{11}^I, \sigma_{22}^I, \sigma_{12}^I)$ in misaligned coordinate system defined by an overall mis-angle;**
- **calculate fiber and matrix true stresses $(\bar{\sigma}_{11}^{f,I}, \bar{\sigma}_{22}^{f,I}, \bar{\sigma}_{12}^{f,I})$ and $(\bar{\sigma}_{11}^{m,I}, \bar{\sigma}_{22}^{m,I}, \bar{\sigma}_{12}^{m,I})$;**
- **calculate the equivalent shear stress through**

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$$\sigma_{12}^{eq} = \frac{(V_f + a_{66}V_m)}{a_{66}K_{12}} \sqrt{\frac{(\bar{\sigma}_{11}^{m,I} + \bar{\sigma}_{22}^{m,I})a - 1 - 2ab}{2a^2}}$$

- **within -90^0< $\theta_c^{f,1}$ <90^0, find out two $\theta_c^{f,1}$ such that one of the resulting functions *f* is positive, whereas other is negative;**
- **bisect the previous interval for a mis-angle increment until a suitable $\theta_c^{f,1}$ is reached so that the function *f* is near to zero.**

(3) Using the obtained $\theta_c^{f,1}$, find out the positive root δ>0 in the physics failure criterion, and amplify the initial σ_{11}<0 with the δ before go to the step (2).

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Repeat applying a modified σ_{11} until δ≈1 is attained, and the resulting σ_{11} is taken as the kinking failure load.

(4) With the final misaligned angle $\theta_c^f = \theta_c^{f,0} + \theta_c^{f,1}$, if the longitudinally applied load σ_{11} in magnitude is higher than the kinking failure load, the composite longitudinal strength is attained by a fiber kinking failure; otherwise, the composite strength is assumed at the fiber failure.

An Excel table based program for finding the misalign-ment angle increment $\theta_c^{f,1}$ subjected to the constraint of δ≈1 has been developed, and can be freely available.

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As an example, let us evaluate the longitudinal compressive strengths of the 9 independent material systems used in three world-wide failure exercises (WWFEs).

Except for the initial fiber misalignment angles, all of the other input parameters are directly taken from the exercise organizers.

Measured strengths specifically longitudinal compressive strengths of the 9 composites have been provided by the exercise organizers as well.

Constituent properties used in WWFEs

	E-Glass LY556	E-Glass MY750	AS4 3501-6	T300 BSL914C	IM7 8511-7	T300 PR319	AS Epoxy	S2-Glass Epoxy	G400-800 5260
Fiber properties									
E_{11}(GPa)	80	74	225	230	276	230	231	87	290
E_{22}(GPa)	80	74	15	15	19	15	15	87	19
ν_{12}	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
G_{12}(GPa)	33.33	30.8	15	15	27	15	15	36.3	27
ν_{23}	0.2	0.2	0.07	0.07	0.36	0.07	0.07	0.2	0.357
$\sigma_{u,t}$(MPa)	2150	2150	3350	2500	5180	2500	3500	2850	5860
$\sigma_{u,c}$(MPa)	1450	1450	2500	2000	3200	2000	3000	2450	3200
Matrix properties									
E(GPa)	3.35	3.35	4.2	4	4.08	0.95	3.2	3.2	3.45
ν	0.35	0.35	0.34	0.35	0.38	0.35	0.35	0.35	0.35
$\sigma_{u,t}$(MPa)	80	80	69	75	99	70	85	73	70
$\sigma_{u,c}$(MPa)	120	120	250	150	130	130	120	120	130
$\sigma_{u,s}$(MPa)	54	54	50	70	57	41	50	52	57
V_f	0.62	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6

An initial fiber misalignment angle has to be assumed.

When zero initial angle, i.e. $\theta_c^{f,0}=0^0$, is assumed, the longitudinal compressive strength is attained always at a fiber failure, with the compressive strength given by

$$\sigma_{11}^c = (V_f + V_m a_{11})\sigma_{u.c}^f$$

Applying this formula to the 9 composites and comparing the predictions with the measured data, the averaged correlation error is 25.1%.

The other predictions are made by taking $\theta_c^{f,0}=1^0$ and 1.5^0.

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Material system	Fiber kinking induced failures (initial misalignment $\theta_c^{f,0}=1^0$)				Fiber strength failure load(MPa)	Longitudinal compression strength		
	Failure load (MPa)	Fiber misaligned angle θ_c^f	failure surface angle θ_n	δ at failure		Measured (MPa)	Predicted (MPa)	Error (%)
E-Glass/LY556	-1300	2.73^0	25.457^0	1.001	924.3	570	924.3	62.16
E-Glass/MY750	-1208	2.839^0	26.37^0	1.0002	898.6	800	898.6	12.33
AS4/3501-6	-2407	2.466^0	33.02^0	1.0005	1521.7	1480	1521.7	2.818
T300/BSL914C	-1950	2.799	12.59^0	1.0005	1217.1	900	1217.1	35.23
IM7/8551-7	-1862	2.558^0	13.93^0	1.0001	1943.1	1590	1862	17.11
T300/PR319	-707	**5.422^0**	13.28^0	0.9999	1214.9	950	707	25.58
AS/Epoxy	-1572	2.679^0	13.98^0	1	1820.9	1500	1572	4.8
S2-Glass/Epoxy	-1302	2.9^0	24.12^0	1.0001	1510.1	1150	1302	13.22
G40-800/5260	-1753	2.79^0	11.26^0	1.0004	1940.2	1700	1753	3.118

$$\text{Averaged error}=\frac{1}{9}\sum_{k=1}^{9} abs(error)_k = 19.6\%$$

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Material system	Fiber kinking induced failures (initial misalignment $\theta_c^{f,0}=1.5^0$)				Fiber strength failure load(MPa)	Longitudinal compression strength		
	Failure load (MPa)	Fiber misaligned angle θ_c^f	failure surface angle θ_n	δ at failure		Measured (MPa)	Predicted (MPa)	Error (%)
E-Glass/LY556	-1190	3.22^0	22.93^0	1.0009	925.1	570	925.1	62.3
E-Glass/MY750	-1115	3.329^0	23.93	1.0008	899.5	800	899.5	12.44
AS4/3501-6	-2044	2.859^0	32.37	1.0001	1522.8	1480	1522.8	2.892
T300/BSL914C	-1689	3.293^0	10.88	1.0003	1218.3	900	1218.3	35.37
IM7/8551-7	-1587	3.048^0	12.15	1.0005	1944.9	1590	1587	0.189
T300/PR319	-648	**5.93^0**	12.97	0.9999	1217.1	950	648	31.79
AS/Epoxy	-1343	3.166	12.45	0.9999	1822.6	1500	1343	10.47
S2-Glass/Epoxy	-1183	3.38	21.81	1.0003	1511.6	1150	1183	2.87
G40-800/5260	-1500	3.286	9.91	1.0005	1942.1	1700	1500	11.76

$$\text{Averaged error}=\frac{1}{9}\sum_{k=1}^{9} abs(error)_k = 18.9\%$$

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The results show that when fiber initial mis-angle attains 1^0, averaged correlation error between the predictions and measurements is reduced from 25.1% at no misalignment to 19.5%, which is within 20%.

If the initial mis-angle is set to 1.5^0, the averaged correlation error is further reduced to 18.9%, which is only a little improvement in accuracy.

According to a classical measurement by Yurgartis in 1980s for initial fiber misalignment angles, more than 80% fibers assumed an initial mis-angle of 1^0.

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Yurgartis claimed that the initial fiber misalignment angle in a composite is not the maximum mis-angle assumed by a single fiber, but is a statistically averaged value.

As such, a practical measurement for an initial fiber mis-angle is not easy to achieve.

The results also demonstrate that even though fiber mis-alignment is taken into account, a limited number of pre-dicted longitudinal compressive strengths show large deviation with the measured counterparts.

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The big correlation error is most likely resulted from an inaccurate measurement for the composite longitudinal compressive strength.

It is well known that the measurement for a longitudinal compressive strength is most difficult among the uniaxial strength data of a UD composite.

Moreover, the longitudinal compressive strength of the composite is also affected by the fiber and matrix compre-ssive strengths, both of which are not easy to measure.

Highlight

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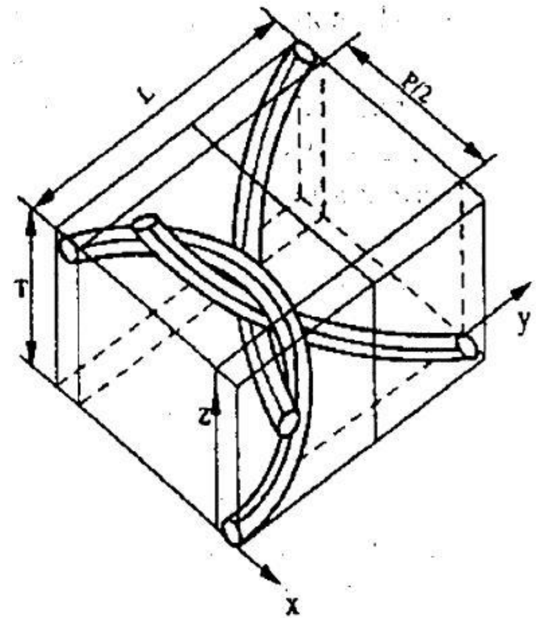
1. Introduction
2. Homogenized stresses
3. True stresses
4. Physics based failure criterion
5. Fiber misalignment angle increment
6. Longitudinal compressive strength
7. **Compressive strength of any composite**
8. Conclusion

For any fibrous composite, its failure and strength prediction can be made as per the following steps.

In 1st step, any composite structure is disretized into a series of brick elements, whose loads can be determined through the overall FEM solution. Each element is loaded uniformly.

In 2nd step, an element is cut to any number of slides, each of which contains at most only straight fiber yarn segments.



7. Compre.-strengt

In 3rd step, each yarn segment in a slide is considered as a UD or uniaxially aligned short fiber composite, whose true stresses in fiber and matrix can be evaluated through Bridging Model and the true stress theory.

If no fiber is in a slide, it is considered as a pure matrix, whose stresses can also be determined by Bridging Model.

In 4th step, the true stresses in fiber and matrix in a local coordinate system are transformed into those in the global one of the element.

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In 5th step, an assemblage of, e.g., iso-strain or iso-stress scheme is applied to determine the overall averaged true stresses in the fiber and matrix of the element.

Based on these true stresses, various failures of the element are detected by respective failure criteria, in which only the original fiber and matrix strengths are needed.

Question arises: how to predict a compressive strength of an element when non-uniaxial fibers such as textile fiber fabrics occur in the element?

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A possible answer is: to adjust the axial compressive strength of the fiber from predicted longitudinal compressive strength of a UD composite from the same material system by taking fiber kinking failure into account.

Then, the calibrated fiber compressive strength can be used to assess a fiber strength failure, without considering fiber misalignment any more.

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8. Conclusion

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Analytical formulae for evaluating a composite longitudinal compressive strength incorporated with fiber kinking failure are presented in this talk.

It is the shear stress component induced from misaligned compression that causes the fiber kinking failure.

A key step is the application of matrix true stress theory. Without the true stresses, the shear component cannot bring matrix to a failure status.

Only the matrix true stresses based on perfect interface bonding are considered in this talk. Future analysis should take an interface debonding into account.

Question & Answer

黄争鸣
huangzm@tongji.edu.cn

Zhou Y., Huang Z.-M., Failure of fiber-reinforced composite laminates under longitudinal compression, *J. Comp. Mater*., 53(24): 3395–3411, 2019. https://doi.org/10.1177/0021998319839217

Huang Z.-M., Mechanics theories for anisotropic or composite materials, *Advances in Applied Mechanics*, 56: 1-137, 2023. https://doi.org/10.1016/bs.aams.2022.09.004